Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics
Question Paper

Wednesday 6 June 2018 – Morning
Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

Model Answers

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h=\dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x)=0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A)+P(B)-P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X=r) = {}^nC_r p^r q^{n-r}$ where $q = 1-p$

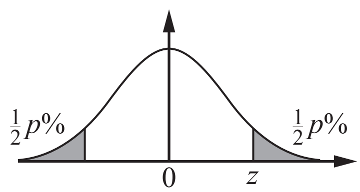
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (23 marks)

1 Show that $(x-2)$ is a factor of $3x^3-8x^2+3x+2$. **[3]**

1 $f(2) = 3(2)^3 - 8(2)^2 + 3(2) + 2$
$= 24 - 32 + 6 + 2$
$= 0$

By factor theorem, $(x-2)$ is a factor

2 By considering a change of sign, show that the equation $e^x-5x^3=0$ has a root between 0 and 1. **[2]**

2 when $x = 0$, $e^x - 5x^3 = e^0 - 0 = 1 > 0$
when $x = 1$, $e^x - 5x^3 = e^1 - 5 = -2.28 < 0$

Change of sign indicates root between 0 and 1

3 **In this question you must show detailed reasoning.**

Solve the equation $\sec^2\theta + 2\tan\theta = 4$ for $0° \leqslant \theta < 360°$. **[4]**

3 $\sec^2\theta + 2\tan\theta = 4$
$1 + \tan^2\theta + 2\tan\theta = 4$
$\tan^2\theta + 2\tan\theta - 3 = 0$
$(\tan\theta + 3)(\tan\theta - 1) = 0$

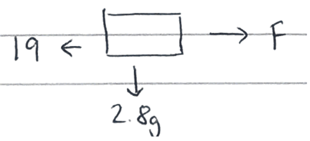
$\tan\theta = -3$ or $\tan\theta = 1$
$\theta = 108.4$ $\theta = 45$
$\theta = 108.4 + 180$ $\theta = 45 + 180$
$= 288.4$ $= 225$

So $\theta = 45, 108.4, 225, 288.4$

4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to $1.2\,\mathrm{m\,s^{-1}}$ as it travels 1.8 m.

(i) Calculate the acceleration of the box. **[2]**

4 i.



$s = 1.8$

$u = 0$ $\quad v^2 = u^2 + 2as$

$v = 1.2$ $\quad 1.2^2 = 0 + 2a(1.8)$

$a = a$ $\quad a = \dfrac{1.2^2}{2 \times 1.8}$

$t = 0$

$a = 0.4\ \mathrm{ms^{-2}}$

(ii) Find the magnitude of the force that Rory applies. **[2]**

ii. $F - 19 = ma$

$F - 19 = 2.8(0.4)$

$F = 1.12 + 19$

$F = 20.12\ \mathrm{N}$

5 The position vector $\mathbf{r}$ metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(i) Find an expression for the velocity of the particle at time t. **[2]**

5 i. $\underline{r} = (1 + 12t - 2t^2)\underline{i} + (t^2 - 6t)\underline{j}$

$\underline{v} = \dfrac{d\underline{r}}{dt} = (12 - 4t)\underline{i} + (2t - 6)\underline{j}$

(ii) Determine whether the particle is ever stationary. **[2]**

ii For the particle to be stationary we need both components to equal zero

At $t = 3$, $12 - 4t = 12 - 4(3) = 0$

$2t - 6 = 2(3) - 6 = 0$

So the particle is stationary at $t = 3$

6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(i) Calculate how much she saves in two years. **[2]**

6 i. This is an arithmetic sequence with $a = 50$, $d = 20$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{24} = \frac{24}{2}(2 \times 50 + 23 \times 20)$$

$$S_{24} = £6720$$

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(ii) Explain why the amounts Baraka saves each month form a geometric sequence. **[1]**

ii. The amount he saves is $1.12 \times$ the amount he saved in the previous month

So this is a geometric sequence with $a = 50$, $r = 1.12$

(iii) Determine whether Baraka saves more in two years than Aleela. **[3]**

iii. For Baraka, $S_{24} = \dfrac{a(r^{24} - 1)}{r - 1}$

$$= \frac{50(1.12^{24} - 1)}{1.12 - 1}$$

$$= £5907.76$$

This is less than the amount Aleela saved

Answer **all** the questions

Section B (77 marks)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

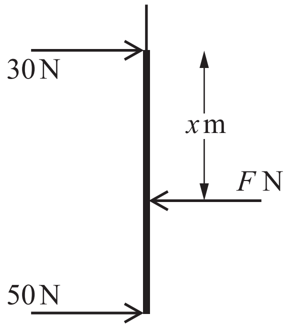


Fig. 7

(i) Find the value of F. **[1]**

7 i $R(\rightarrow)$: $30 + 50 = F$

$F = 80\text{N}$

(ii) Find the value of x. **[2]**

ii. Take moments about the top of the rod

$Fx = 50(2)$

$80x = 100$

$x = 1.25$

8 **(i)** Show that $8\sin^2 x\cos^2 x$ can be written as $1 - \cos 4x$. **[3]**

8 i. $8\sin^2 x\cos^2 x = 2(1 - \cos 2x)(1 + \cos 2x)$

$= 2(1 - \cos^2 2x)$

$= 2 - 2\cos^2 2x$

$= 2 - (\cos 4x + 1)$

$= 1 - \cos 4x$

(ii) Hence find $\int \sin^2 x \cos^2 x \, dx$. **[3]**

ii. $\int \sin^2 x \cos^2 x \, dx = \frac{1}{8} \int 1 - \cos 4x \, dx$

$= \frac{1}{8} \left[x - \frac{1}{4} \sin 4x \right] + c$

$= \frac{x}{8} - \frac{1}{32} \sin 4x + c$

9 A pebble is thrown horizontally at $14\,\mathrm{m\,s^{-1}}$ from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high d m away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

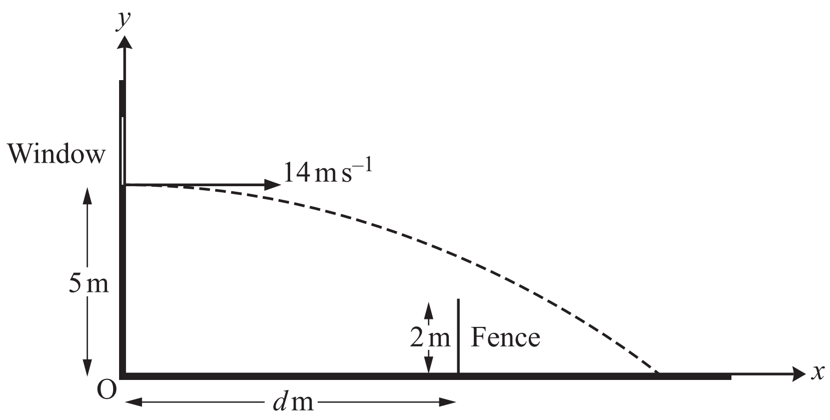


Fig. 9

(i) Find the time the pebble takes to reach the ground. **[3]**

9 i. $\uparrow$: $s = -5$

$u = 0$

$v = -$

$a = -9.8$

$t = t$

$s = ut + \frac{1}{2}at^2$

$-5 = 0 + \frac{1}{2}(-9.8)t^2$

$t^2 = \frac{10}{9.8}$

$t = 1.01$ s

(ii) Find the cartesian equation of the trajectory of the pebble. **[4]**

ii. $\rightarrow$: speed = $\frac{\text{distance}}{\text{time}}$

distance = speed x time

$x = 14t$

$\uparrow$: $s = ut + \frac{1}{2}at^2$

$y = 0(t) + \frac{1}{2}(-9.8)t^2$

$y = -4.9t^2$

This is the vertical height below the window. The distance above the ground is an extra 5m than this.

$y = 5 - 4.9t^2$

So the Cartesian equation is

$y = 5 - 4.9\left(\frac{x}{14}\right)^2$

$y = 5 - \frac{x^2}{40}$

(iii) Find the range of possible values for *d*. **[3]**

iii. when $y = 2$: $2 = 5 - \frac{x^2}{40}$

$\frac{x^2}{40} = 3$

$x^2 = 120$

$x = 10.954..$

$\therefore$ $0 < d < 11$

10 Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

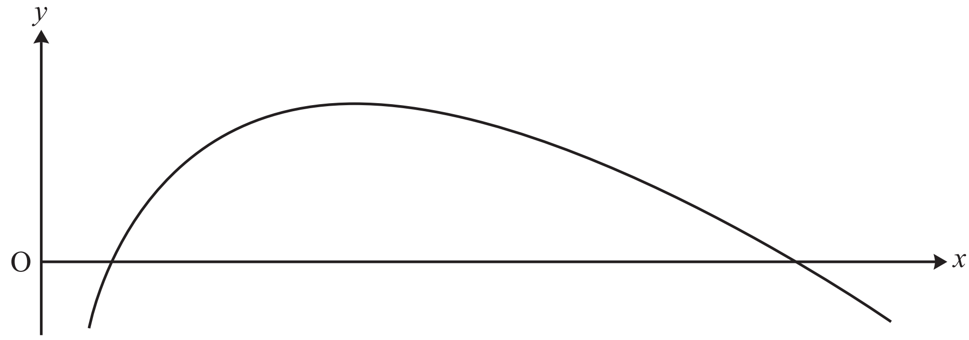


Fig. 10

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

10. First find where the curve crosses the x axis

At $y = 0$: $0 = (k - x)\ln x$

$k - x = 0$ or $\ln x = 0$

$x = k$ $\quad$ $x = 1$

So the first point it crosses at is $x = 1$, and the second point is $x = k$

$$\text{Area} = \int_1^k (k - x)\ln x \, dx$$

let $u = \ln x$ $\quad$ $\frac{dv}{dx} = k - x$

$\frac{du}{dx} = \frac{1}{x}$

$v = kx - \frac{1}{2}x^2$

Integration by parts

$$\text{Area} = \left[\ln x \left(kx - \tfrac{1}{2}x^2\right)\right]_1^k - \int_1^k \frac{1}{x}\left(kx - \tfrac{1}{2}x^2\right) dx$$

$$= \left[\ln k\left(k^2 - \tfrac{1}{2}k^2\right) - 0\right] - \int_1^k k - \tfrac{1}{2}x \; dx$$

$$= \tfrac{1}{2}k^2 \ln k - \left[kx - \tfrac{1}{4}x^2\right]_1^k$$

$$= \tfrac{1}{2}k^2 \ln k - \left[k^2 - \tfrac{1}{4}k^2 - k + \tfrac{1}{4}\right]$$

$$= \tfrac{1}{2}k^2 \ln k - \tfrac{3}{4}k^2 + k - \tfrac{1}{4}$$

11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

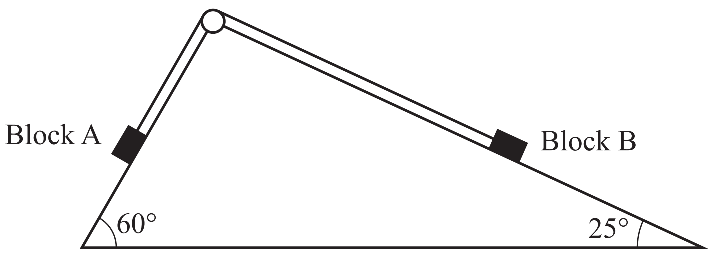
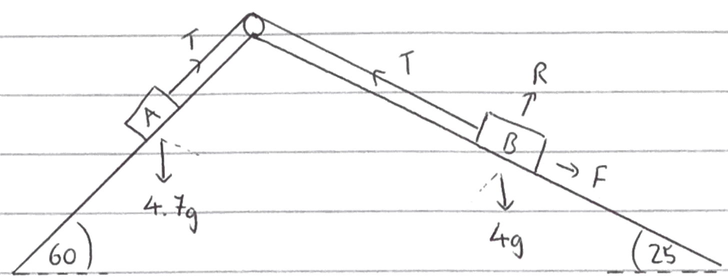


Fig. 11

(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. **[2]**

11 i.



A is in equilibrium so

$T = 4.7g \sin 60$

$T = 39.889\ldots$

$T = 39.9\text{ N}$

(ii) Find the coefficient of friction between the rough plane and Block B. **[5]**

ii. Resolve the forces on B

$T - 4g\sin 25 - F = 0$

$F = 39.9 - 4g\sin 25$

$\mu R = 39.9 - 4g\sin 25$

$\mu = \dfrac{39.9 - 4g\sin 25}{4g\cos 25}$

$\mu = 0.656$

12 Fig. 12 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

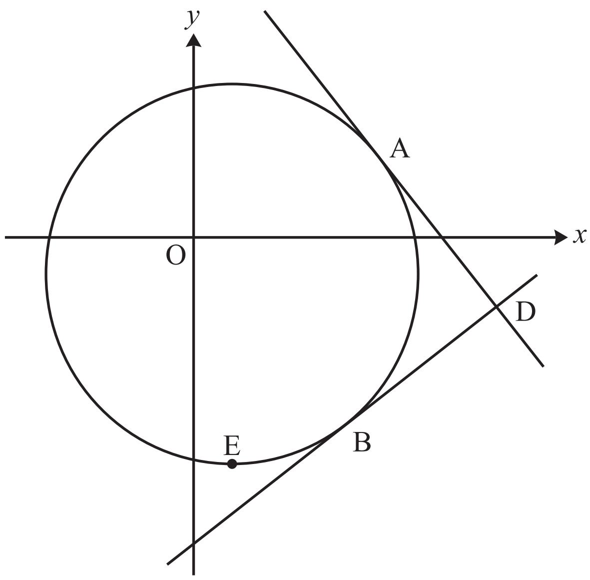


Fig. 12

(i) Write down the coordinates of C, the centre of the circle. **[1]**

12 i) $(1, -1)$

(ii) (A) Show that the line 43yx=- 32 is a tangent to the circle. **[4]**

ii. A) $y = \frac{3}{4}x - 8$

sub this into the equation of the circle

$(x-1)^2 + (\frac{3}{4}x - 8 + 1)^2 = 25$

$x^2 - 2x + 1 + (\frac{3}{4}x - 7)^2 = 25$

$x^2 - 2x + 1 + \frac{9}{16}x^2 - \frac{21}{2}x + 49 = 25$

$\frac{25}{16}x^2 - \frac{25x}{2} + 25 = 0$

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

There is only one solution to this equation meaning the line and the circle only meet at one point

Therefore the line is a tangent

(B) Find the coordinates of B, the point where the line 43yx=-32 touches the circle. **[1]**

B) $x = 4$ from before

when $x = 4$, $y = \frac{3}{4}(4) - \frac{32}{4}$

$y = -5$

So B is $(4, -5)$

(iii) Prove that ADBC is a square. **[3]**

iii The tangents are perpendicular to the circle, radius, so $\angle CAD = 90°$ and $\angle CBD = 90°$

Gradient AC $= \frac{2 - -1}{5 - 1} = \frac{3}{4}$

Gradient BC $= \frac{-1 - -5}{1 - 4} = \frac{-4}{3}$

So BC and AC are perpendicular

This means $\angle ACB = 90°$

$\angle ADB = 90°$ because the angles add up to 360

So ACBD is a rectangle

$AC = AB = 5$ (radius)

So all the sides have length 5

$\therefore$ ADBC is a square

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. **[5]**

iv. When y is at its lowest, $(y+1)^2$ is at its max

$(y+1)^2 = 25 - (x-1)^2$

This is largest when $(x-1)^2 = 0$, so $x = 1$

$(y+1)^2 = 25$

$y + 1 = -5$

$y = -6$

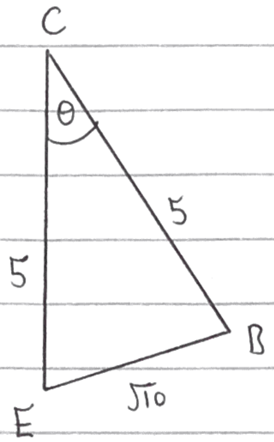
So E has coordinates $(1, -6)$

$BE^2 = (4-1)^2 + (-5--6)^2$

$= 3^2 + 1^2$

$= 10$

$BE = \sqrt{10}$



cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$\sqrt{10}^2 = 5^2 + 5^2 - 2(5)(5)\cos\theta$

$\cos\theta = \dfrac{5^2 + 5^2 - 10}{2 \times 25}$

$\cos\theta = 0.8$

$\theta = 0.6435^c$

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 5^2 \times 0.6435$

$= 8.04$

13 The function f(x) is defined by $f(x) = \sqrt[3]{27-8x^3}$. Jenny uses her scientific calculator to create a table of values for f(x) and f$'(x)$.

x	f(x)	f$'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for f$'(x)$ and hence explain why the calculator gives an error for f$'(1.5)$. **[3]**

13 i. $f(x) = \sqrt[3]{27 - 8x^3} = (27 - 8x^3)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}(-8x^2)(3)(27 - 8x^3)^{-\frac{2}{3}}$

$= \dfrac{-8x^2}{(27 - 8x^3)^{\frac{2}{3}}}$

At $x = 1.5$, $f'(1.5) = \dfrac{-8(1.5)^2}{(27 - 27)^{\frac{2}{3}}}$

$= \dfrac{-8(1.5)^2}{0}$

The denominator is zero so the result is undefined

(ii) Find the first three terms of the binomial expansion of f(x). **[3]**

ii. $f(x) = (27 - 8x^3)^{\frac{1}{3}}$

$= \left[27\left(1 - \frac{8}{27}x^3\right)\right]^{\frac{1}{3}}$

$= 3\left(1 - \frac{8}{27}x^3\right)^{\frac{1}{3}}$

$= 3\left(1 + \left(\frac{1}{3}\right)\left(-\frac{8}{27}x^3\right) + \dfrac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{8}{27}x^3\right)^2}{2!}\right)$

$$= 3\left(1 - \frac{8}{81}x^3 - \frac{64}{6561}x^6\right)$$

$$= 3 - \frac{8x^3}{27} - \frac{64x^6}{2187}$$

(iii) Jenny integrates the first three terms of the binomial expansion of $\mathrm{f}(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3}\,\mathrm{d}x$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

iii. The expansion is valid for $\left|\frac{-8}{27}x^3\right| < 1$

$|x^3| < \frac{27}{8}$

$|x| < \frac{3}{2}$

The limits 0 and 1 of the integral both satisfy this condition so her method is valid.

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3}\,\mathrm{d}x$. **[3]**

iv. let $f(x) = \sqrt[3]{27-8x^3}$

$f(0) = 3$

$f(0.25) = 2.9954$

$f(0.5) = 2.9625$

$f(0.75) = 2.8694$

$f(1) = 2.6684$

$$I \approx \frac{0.25}{2}\left(3 + 2.6684 + 2(2.9954 + 2.8694 + 2.9625)\right)$$

$$= 2.915$$

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. The graph of $y = f(x)$ is shown in Fig. 13.

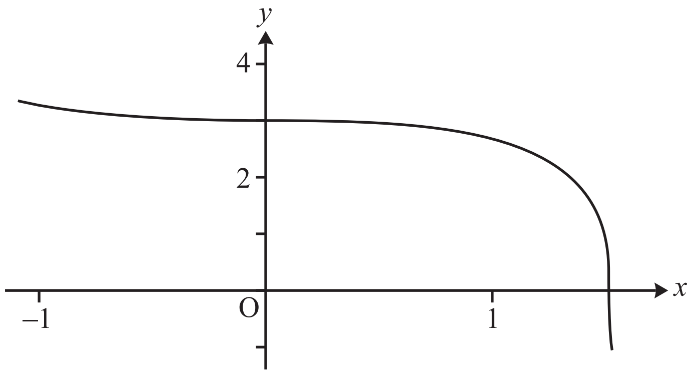


Fig. 13

(v) Explain why the trapezium rule gives an underestimate. **[1]**

v. The curve slopes outwards, so the area between the top of the trapezia and the curve is missing from the estimate

14 The velocity of a car, $v\,\text{m}\,\text{s}^{-1}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\text{m}\,\text{s}^{-1}$ and it accelerates to $11.4\,\text{m}\,\text{s}^{-1}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. **[3]**

14 i.

$s = -$

$u = 5$ $\quad v = u + at$

$v = 11.4$ $\quad 11.4 = 5 + 4a$

$a = a$ $\quad 4a = 6.4$

$t = 4$ $\quad a = 1.6$

Now we know the acceleration we can find an equation for a general time t

$s = -$

$u = 5$

$v = v$ $\quad v = u + at$

$a = 1.6$ $\quad v = 5 + 1.6t$

$t = t$

(ii) Explain why this model is not appropriate in the long term. **[1]**

ii. The car cannot accelerate forever, it will have a maximum velocity

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\text{m s}^{-1}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. **[3]**

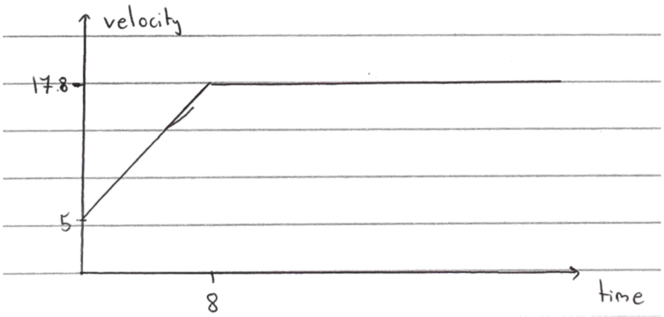
iii. when $v = 17.8$:

$17.8 = 5 + 1.6t$

$1.6t = 12.8$

$t = 8$

So the car stops accelerating after 8 seconds



(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. **[3]**

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

iv. Displacement = Area under the curve

$= 5 \times 8 + \dfrac{8 \times (17.8 - 5)}{2} + (20-8) \times 17.8$

$= 40 + 51.2 + 213.6$

$= 304.8\,\text{m}$

(v) Show that this model gives an appropriate value for v when $t = 4$. **[1]**

v. when $t = 4$, $v = 5 + 0.6(4)^2 - 0.05(4)^3$

$= 5 + 9.6 - 3.2$

$= 11.4$

This matches the value we were given earlier

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. **[3]**

vi. $a = \frac{dv}{dt} = 0.6 \times 2t - 0.05 \times 3t^2$

$= 1.2t - 0.15t^2$

when $t = 8$, $a = 1.2(8) - 0.15(8)^2$

$= 0$

This is a better model because it doesn't require a sudden change in acceleration which happens in model A.

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

vii. $r = \int_0^8 v = \int_0^8 5 + 0.6t^2 - 0.05t^3 \, dt$

$= \left[5t + 0.2t^3 - 0.0125t^4\right]_0^8$

$= \underline{91.2 \text{ m}}$

Model A gave a displacement of $40 + 51.2$

$= 91.2$ m in the first 8 seconds

Displacement is the same for the next 12 seconds so the total displacement after 20 seconds is the same as in model A

END OF QUESTION PAPER

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